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# Random lattice strings vs. type IIB matrix models

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## Abstract

I comment on a curious relation between Siegel's model of random lattice strings and type IIB matrix model. The comparison of the two theories suggests that there may exist extra terms in the latter which are overlooked in the weak string coupling limit.

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Various matrix models [1],[2],[3] which are now extensively discussed in the context of non-perturbative string theory do not necessarily refer to discretization of the world sheet. Matrices in these theories emerge as images of non-commuting coordinates associated with either D-particles [1] or D-instantons [2],[3]. In other words, the integration over random matrices in these models is not intending to replace the summation over world sheet topologies. At the same time the random matrix approach based on the idea of a discretization of the world sheet has proved remarkably successful in understanding the non-perturbative structure of string theories in dimensions less than two [4]. Therefore, it might be interesting to look for some possible interplay between new and old matrix model approaches. One nontrivial example of such a relation has already been discussed in [5].

In the present letter, I would like to focus on one remarkable fact about random lattice strings which may turn out to be relevant in current attempts to construct a non-perturbative string theory. It has been observed that planar Feynman diagrams of matrix models have a duality symmetry under replacing vertices with loops and vice versa [6],[7]. In [8] this symmetry of random lattices has been connected to the T-duality of long- and short-distance behavior in string theory [9]. Namely, the duality invariance of planar Feynman diagrams gives rise to the T-duality invariance of string perturbation theory.

Siegel was the first to realize that the T-duality of the continuum string in its random lattice representation uniquely determines the random matrix model potential [10]. His approach has further been generalized in [11] to include self-dual matter systems.

Siegel's T-self-dual matrix model is given as follows [10]

$$S_\Phi = \text{tr} \left[ \frac{1}{2} \Phi^2 + N \ln(1 - g\Phi) \right], \quad (1)$$

where  $\Phi$  is a hermitian  $N \times N$  matrix and  $g$  is a constant. This model can be thought of as describing a  $D = 0$  string.

Let us make the following change of variables

$$1 - g\Phi = gY. \quad (2)$$

In terms of  $Y$  eq.(1) takes the following form

$$S_Y = \text{tr} \left[ -\frac{Y}{g} + N \ln Y + \frac{Y^2}{2} + \frac{1}{2g^2} + N \ln g \right]. \quad (3)$$

In the limit  $g \rightarrow 0$ ,  $N \rightarrow \infty$  eq.(3) goes to

$$S_Y(g \rightarrow 0, N \rightarrow \infty) = \text{tr} \left[ -\frac{Y}{g} + N \ln Y + \text{constant} \right]. \quad (4)$$

The constant term is irrelevant for the duality property. The limit  $g \rightarrow 0$ ,  $N \rightarrow \infty$  is taken so that  $g \cdot N$  is kept fixed.

The curious fact is that, up to a normalization of the constants  $g$  and  $N$ , eq.(4) coincides with the saddle point of the type IIB matrix model [5]. According to ref.[3], the constant  $g$  in front of the linear in  $Y$  term is proportional to the string coupling constant,  $g \sim g_s$ . Therefore, the matrix action given by eq.(4) can be considered as a weak string coupling limit of the Siegel matrix model. In particular, the latter has to contain type IIB D-branes which are observed among solutions of the IIB matrix model [2],[3].

However, if we take the Siegel model as a definition of the exact type IIB matrix theory at the saddle point, then the  $Y^2$ -term in eq.(3) becomes important at finite values of the string coupling constant. It is natural to assume that the duality invariance of the Siegel random matrix model is an underlying symmetry of string perturbation theory and as such has to be preserved in any non-perturbative formulation. Bearing this principle in mind, one can write down a modified type IIB matrix model. Namely,

$$S_{IIB} = \alpha \text{tr} \left\{ -\frac{1}{4} Y^{-1} [A_\mu, A_\nu]^2 - \frac{1}{2} (\bar{\psi} \Gamma^\mu [A_\mu, \psi]) \right\} + \text{tr} [\beta Y + \gamma \ln Y + \sigma Y^2 + \delta]. \quad (5)$$

Here  $A_\mu$  and  $\psi_\alpha$  are  $N \times N$  hermitian bosonic and fermionic matrices respectively. This theory possesses the N=2 supersymmetry in the limit  $N \rightarrow \infty$  [3]. The constant parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\sigma$ ,  $\delta$  get, in general, renormalized. The hope is that they run to the values predicted by the T-duality. This will be studied elsewhere.

To conclude, the Siegel random lattice string and the type IIB matrix model have quite different setups. However, both theories aim at one and the same target - a non-perturbative description of string theory (whatever it might be). Therefore, it is not very surprising if the two approaches eventually will merge.

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